Aerial Power Cables Profile, Sag and Tension Calculations

Jorge R. López, MSME, PE
José A. López, MSCE, PE

Introduction

With the events of Hurricane Hugo, Georges and others, the design of aerial power cables took a lot of interest. This mainly because concrete power poles designed to withstand cable loads, failed and caused significant damage to the electrical distribution system in Puerto Rico. Rigorous testing of the reinforced concrete power poles manufactured in Puerto Rico was the obvious step in determining the causes of the pole failures. The conclusion of the testing was that the power poles were designed and manufactured correctly. The only other cause for the power pole failures was that power cables installed were over loading the pole capacities. In order to improve the knowledge of professional engineers in Puerto Rico, the Puerto Rico Electric Power Authority (PREPA) provided guidance material in a technical seminar. The aim of the seminar was to provide criteria for determining the loads power poles were subjected. The first seminar was provided in September of 2000.

Although the material presented in the seminar is complete, further review of it has shown that the calculations presented can be extended to more general cases and with more accuracy. The use of numerical methods is required to accomplish this.

Flexible Cables

The formulation to determine the profile and tension of a suspended flexible cable relies on the assumption that a cable offers no resistance to bending. The two most common load distributions on a cable are a load uniformly distributed along the span and a load distributed uniformly along the length of the cable. When a cable is loaded uniformly along its span it forms in the shape of a parabola and when it is loaded uniformly along its length it forms in the shape of a catenary. Both loading conditions occur in a suspended cable that is subject to wind. One of the most common properties studied in each case is the sag. Sag can be defined in two ways, as the difference in elevation between the lowest point on the cable and a support (see Figure 1-A), and as the difference in elevation between the point where the cable profile slope is equal to the slope between the start and end of the cable (See Figure 1-B). For the purpose of this article the later definition (Figure 1-B) will be used.

![Figure 1 - Sag Definitions](image)
Weight Loading
When a flexible cable is loaded by its weight, the load is distributed uniformly along the cable length. The cable will form in the shape of a catenary. The general equation of the catenary curve in Cartesian coordinates is as follows:\(^1\):

\[
y = C \cosh \left( \frac{x-H}{C} \right) + V
\]

Equation 1

\[
C = \frac{T_D}{q}
\]

Equation 2

\[
T_D = \text{Cable Tension at Lowest Curve Point (Lb)}
\]

\[
q = \text{Cable Weight (Lb/Ft)}
\]

The cable tension is a parameter that has to be assumed and iterated to provide the required sag. Figure 2 shows the equation parameters and variables important to the catenary problem. The axes are located at the cable starting point. The horizontal span is \(\Delta x\) and the vertical span is \(\Delta y\). \(V\) is the vertical offset of the curve vertex (lowest point) from x axis and \(H\) is the horizontal offset of the vertex from the y axis.

\[
\Delta x = C \cosh \left( \frac{\Delta x - H}{C} \right) - C \cosh \left( \frac{-H}{C} \right)
\]

Equation 3

\[
\Delta y = C \cosh \left( \frac{x-H}{C} \right) - C \cosh \left( \frac{-H}{C} \right)
\]

Equation 4

To find the value of the horizontal offset \(H\) equation 4 has to be iterated. This can be simply accomplished by using the bisection method.

The tension in the catenary cable is given by\(^1\):

\[
T = T_D \cosh \left( \frac{x-H}{C} \right)
\]

Equation 5
Thus, equation 5 can be evaluated at the cable start ($x=0$) and the cable end ($x=\Delta x$) to calculate the tension at each support. The angle at which the tensions are directed can be found by differentiating Equation 1 to find the slope of the curve and evaluate it at the cable start and end. The slope of the curve ($m$) and angle ($\theta$) are given by:

$$m = \sinh\left(\frac{x-H}{C}\right)$$  \hspace{1cm} \text{Equation 6}

$$\theta = \tan^{-1}(m)$$  \hspace{1cm} \text{Equation 7}

Finally, the location where the sag occurs is where the slope of the curve is equal to the slope of the line that goes from cable start to cable end. This is given by:

$$\frac{\Delta y}{\Delta x} = \sinh\left(\frac{x_{SAG}-H}{C}\right)$$  \hspace{1cm} \text{Equation 8}

Again, in order to find the location of the sag ($x_{SAG}$) one could use an iterative method such as the bisection method. Once the location is determined, the sag is calculated by subtracting the elevation of the line from the elevation of the curve. The sag is given by:

$$SAG = \frac{\Delta y}{\Delta x} x_{SAG} - C \cosh\left(\frac{x_{SAG}-H}{C}\right) - V$$  \hspace{1cm} \text{Equation 9}

In order to calculate the length of the cable, one can use the following integral$^1$:

$$L = \int_0^{\Delta x} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$  \hspace{1cm} \text{Equation 10}

Substituting equation 6 into equation 10 one can find that the length is given by:

$$L = C \left(\sinh\left(\frac{\Delta x-H}{C}\right) - \sinh\left(-\frac{H}{C}\right)\right)$$  \hspace{1cm} \text{Equation 11}

**Parabolic Approximation**

The equations presented to solve the catenary problem make use of hyperbolic functions. These functions do not render themselves easy to find direct solutions. A common approximation to the catenary is the use of the solution given for a load that is uniformly distributed along the span namely, the parabolic approximation. The use of this approximation relies on the assumption that when the sag is small compared to the span, the weight of the cable behaves similar to a load distributed uniformly along the span. The general equation of the parabolic curve in Cartesian coordinates is as follows$^1$:
\[ y = K(x - H)^2 + V \]  
Equation 12

\[ K = \frac{q}{2T_D} \]  
Equation 13

\[ T_D = \text{Cable Tension at Lowest Curve Point (Lb)} \]
\[ q = \text{Cable Weight (Lb/Ft)} \]

Using the coordinate system in Figure 2, the values for the two axes offset can be defined as follows:

\[ V = -KH^2 \]  
Equation 14

\[ H = \frac{K\Delta x^2 - \Delta y}{2K\Delta x} \]  
Equation 15

A more usable form of Equation 12 is given if we substitute the value of the horizontal offset \(H\):

\[ y = K(x - 2H) \]  
Equation 16

The tension in the parabolic cable is given by\textsuperscript{1}:

\[ T = \sqrt{T_D^2 + q^2(x - H)^2} \]  
Equation 17

Substituting the value of \(q\) based on equation 13 we obtain:

\[ T = T_D \sqrt{1 + 4K^2(x - H)^2} \]  
Equation 18

Thus, equation 18 can be evaluated at the cable start \((x=0)\) and the cable end \((x=\Delta x)\) to calculate the tension at each support. By doing this we obtain:

\[ T_{x=0} = T_D \sqrt{1 + \left(\frac{\Delta y}{\Delta x} \right)^2} \]  
Equation 19

\[ T_{x=\Delta x} = T_D \sqrt{1 + \left(\frac{\Delta y}{\Delta x} \right)^2} \]  
Equation 20

The angle at which the tensions are directed can be found by differentiating Equation 12 to find the slope of the curve. The slope of the curve \((m)\) and angle \((\theta)\) are given by:

\[ m = 2K(x - H) \]  
Equation 21

\[ \theta = \tan^{-1}(m) \]  
Equation 22
Thus, equation 21 and 22 can be evaluated at the cable start \((x=0)\) and the cable end \((x=\Delta x)\) to calculate the angle of the tension at each support. By doing this we obtain:

\[
\theta_{x=0} = \tan^{-1}\left( K\Delta x^2 - \frac{\Delta y}{\Delta x} \right) \quad \text{Equation 23}
\]

\[
\theta_{x=\Delta x} = \tan^{-1}\left( K\Delta x^2 + \frac{\Delta y}{\Delta x} \right) \quad \text{Equation 24}
\]

It can also be shown that the tension of the line is equal to:

\[
T = T_D \sec(\theta) \quad \text{Equation 25}
\]

Finally, the location where the sag occurs is where the slope of the curve is equal to the slope of the line that goes from cable start to cable end. This is given by:

\[
\frac{\Delta y}{\Delta x} = 2K(x_{sAG} - H) \quad \text{Equation 26}
\]

Substituting the value of the horizontal offset \(H\) found in equation 15 into equation 26 we obtain where the sag occurs in a more usable form:

\[
x_{sAG} = \frac{\Delta x}{2} \quad \text{Equation 27}
\]

The sag is calculated by subtracting the elevation of the line from the elevation of the curve. The sag is given by:

\[
SAG = \frac{\Delta y}{2} - K \frac{\Delta x}{2} \left( \frac{\Delta x}{2} - \frac{(K\Delta x^2 - \Delta y)}{K\Delta x} \right) \quad \text{Equation 28}
\]

If equation 28 is reduced to its minimum form you can find a more usable form of the sag:

\[
SAG = \frac{K\Delta x^2}{4} \quad \text{Equation 29}
\]

Sometimes it is useful to define the value of the parabolic factor \(K\) based on the sag and is given resolving equation 29 for \(K\) as follows:

\[
K = \frac{4SAG}{\Delta x^2} \quad \text{Equation 30}
\]

In the same way if we substitute the value of \(K\) as defined in equation 13 in equation 30 and solve for the minimum tension \(T_D\) we obtain:
\[ T_D = \frac{q\Delta x^2}{8SAG} \]  

Equation 31

The parabolic cable length is given by substituting equation 21 into equation 10 and is given by:

\[ L = \frac{1}{4K} \left( u\sqrt{u^2 + 1} + \ln\left( u + \sqrt{u^2 + 1} \right) \right)^{2K(\Delta x - H)} \]  

Equation 32

Now that we have derived all required equations to approximate the catenary, let’s look at the error induced by the parabolic approximation. Figure 3 shows the profiles of the catenary and the parabolic solution for a case where the vertical offset between the start and end of the cable has been set to zero (\( \Delta y=0 \)) for a span of 1000 Ft, and cable weight of 5 Lb/Ft and the lowest tension is 250 Lb. As it can be seen, the catenary cable hangs below the parabolic cable making the sag significantly different for the presented solutions. As the cable tension is increased and the sag gets smaller, the cable gets to be closer to a line and both solutions converge to close values.

Figure 3 – Catenary and Parabolic Cable Profile for 0% Slope (\( T_D=250 \) Lb, \( q=5 \) Lb/Ft)

Figure 4 shows what happens to the error between the parabolic approximation and the catenary. The figure shows the error in the sag, tension and horizontal force using the catenary as the base. As it can be seen, the sag error gets below 0.5% after the sag ratio is greater than 20 (\( \Delta x/Sag>20 \))
or 5% sag. With the tension, the error gets below 5% after the sag ratio is greater than 35 \((\Delta x/\text{Sag}>35)\) or 2.86% sag. With the X-Axis component of the tension \((F_x)\), the error gets below 3% after the sag ratio is greater than 35.

![Graph](attachment:image.png)

**Figure 4 - Parabolic Sag and Tension Error for 0% Slope**

It is important to note that this is the error for the case where the vertical offset between the start and end of the cable is zero (0% Slope). Figure 5 shows the profiles of the catenary and the parabolic solution for a case where the vertical offset between the start and end of the cable has been set to 15\% \((\Delta y/\Delta x = 0.15)\) for a span of 1000 Ft, and cable weight of 5 Lb/Ft and the lowest tension is 250 Lb. Again, the catenary cable hangs below the parabolic cable making the sag significantly different for the presented solutions.

Figure 6 shows what happens to the error between the parabolic approximation and the catenary. The figure shows the error in the sag, tension and horizontal force using the catenary as the base. As it can be seen, the sag error gets below 2\% after the sag ratio is greater than 12 \((\Delta x/\text{Sag}>12)\) or 8.3\% sag. With the tension, the error **does not** gets below 7\% even after sag ratios greater than 80 \((\Delta x/\text{Sag}>80)\) or 1.25\% sag. With the X-Axis component of the tension \((F_x)\), the error **does not** gets below 9.0\% even after sag ratios greater than 80. The implication of this is clear, with significant cable slopes; the catenary solution should be used.
Figure 5 - Catenary and Parabolic Cable Profile for 15% Slope ($T_D=250$ Lb, $q=5$ Lb/Ft)

Figure 6 - Catenary and Parabolic Cable Profile for 15% Slope ($T_D=250$ Lb, $q=5$ Lb/Ft)
Temperature Effect

A hanging flexible cable that is subjected to a change in temperature changes its length. An increase in temperature will increase the cable length, will decrease the cable linear weight and will decrease the cable tension. A decrease in temperature will decrease the cable length, will increase the cable linear weight and will increase the cable tension. The effect of temperature on the cable length can be predicted by means of the cable coefficient of expansion. Using the coefficient of expansion, we obtain the following:

\[ L_f = L_i \left(1 + \alpha(T_f - T_i)\right) \]

\[ \alpha = \text{coefficient of expansion (1/°F)} \]
\[ L_i = \text{Final Length (Ft)} \]
\[ L_o = \text{Initial Length (Ft)} \]
\[ T_i = \text{Final Temperature (°F)} \]
\[ T_o = \text{Initial Temperature (°F)} \]

Having calculated the new length, equations 32 or 11 depending on the solution selected, can be used to calculate the required Lowest Tension \( (T_d) \) that will make the cable length equal to the temperature changed length. This calculation requires an iterative solution such as the Bi-Section Method. With the calculated \( T_d \) and the problem load \( (q) \) and the horizontal and vertical span lengths \( (\Delta x \text{ and } \Delta y \text{ or } \Delta l) \); the factors \( (K \text{ or } C) \), offsets \( (H \text{ and } V \text{ or } D) \), sag, tensions and angles can be calculated using the equations provided in this article.

Wind Loading

When a flexible cable is loaded by wind pressure, the load is distributed uniformly along the cable span and the cable forms a parabolic shape in the wind plane. Care must be taken how the span is defined. On cables whose start and end are closely at the same elevation, the horizontal span is the correct definition. But in the case of a cable whose start and end are at different heights, i.e. with a significant angle from horizontal \( (\gamma) \). The span \( (\Delta l) \) is composed by the horizontal component and the vertical component and the correct length and angle to use would be:

\[ \Delta l = \sqrt{\Delta x^2 + \Delta y^2} \]

\[ \gamma = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) \]

Another condition of wind loading is that because the maximum loads occur with the wind perpendicular to the cable angle, there is no actual offset in the start and end of the cable with respect to the wind direction. In this case, the parabolic shape to be formed will be symmetric. This makes the tension and angle on both supports equal. The case of wind loading a cable at an
angle is shown on Figure 7. Wind pressures shall be as per ASCE7-95. The wind load \( q_{\text{WIND}} \) is given by:

\[
q_{\text{WIND}} = P_{\text{wind}} \frac{d_c}{12} \quad \text{(Lb/ft)}
\]

Equation 36

\[
P_{\text{WIND}} = \text{Wind Pressure (Lb/ft}^2\text{)}
\]

\[
d_c = \text{Diameter of Conductor (in)}
\]

The equations derived previously for the parabolic approximation can be used for wind loading and will not incur in any error since the wind pressure will shape the cable in the form of a parabola. For this we will have to use these equations using \( \Delta y=0 \) and \( \Delta l \) instead of \( \Delta x \).

Using these changes, the parabolic factor \( K \) is defined to be:

\[
K_{\text{WIND}} = \frac{4SAG_{\text{WIND}}}{\Delta l^2}
\]

Equation 37

And the Lowest Tension \( T_d \) is defined to be:

\[
T_{d,\text{WIND}} = \frac{q_{\text{WIND}}\Delta l^2}{8SAG_{\text{WIND}}}
\]

Equation 38

Figure 7 – Wind loading of a cable that lies at an angle
The horizontal and vertical offsets are now to be interpreted to be in a direction perpendicular to the wind. In order to show this transformation, the vertical offset \( (V) \) will be called the depth offset \( (D) \). Using equation 14 and the transformed definition of the parabolic factor \( (K) \) in equation 37, the depth offset is defined as:

\[
D = -SAG_{\text{WIND}} \quad \text{Equation 39}
\]

Using equation 15, the horizontal offset is defined as:

\[
H_{\text{WIND}} = \frac{\Delta l}{2} \quad \text{Equation 40}
\]

Using equations 19 and 20, the tension in the cable at the supports is defined as:

\[
T_{\text{WIND}} = T_{D,\text{WIND}} \sqrt{1 + \frac{16SAG_{\text{WIND}}^2}{\Delta l^2}} \quad \text{Equation 41}
\]

Using equation 23 and 24 the angle of the tension is:

\[
\theta_{\text{WIND}} = \tan^{-1}(4SAG_{\text{WIND}}) \quad \text{Equation 42}
\]

Using equation 32 and 40 the length of the cable is:

\[
L = \frac{1}{4K} \left( u \sqrt{u^2 + 1} + \ln \left( u + \sqrt{u^2 + 1} \right) \right)^{KN}_{-K\Delta l} \quad \text{Equation 43}
\]

When finding the solution to the parabolic cable for the wind load, care must be taken as the length of the cable using the parabolic solution for wind has to match the cable length for either, the catenary or parabolic solution for weight. In this regard, the importance of using the correct length for the parabolic solution for wind can be seen. If instead of using the angled span length \( (\Delta l) \), the horizontal span length is used \( (\Delta x) \), the error generated in the calculated cable tensions due to the wind are significant and increases with the slope of the cable. It is interesting to note that the error in the lateral wind load would be insignificant (1% error for a 10% slope). Figure 8 shows how the error of the tension increases with the slope for a symmetric parabolic case such as wind loading. As it can be seen, for a 5% slope cable with 5% sag, the error is close to 10%.
The reason why the increase in slope increases the error in the tension can be seen on Figure 9. A small increase in length ratio (ie. selecting $\Delta l$ for span length rather than $\Delta x$) will require a large decrease in sag to maintain the length. This in turn causes an increase in tension. The effect of the sag change upon the tension depends on the location on the Sag-Tension curve. From Figure 9 for sags greater than 6%, the effect on increasing the tension is reduced, for sag less than 6% the effect on increasing the tension is amplified. Since the parabolic approximation for weight loading is usually used for sag less than 5%, the lateral sag required for the wind loading will be less than 5% to maintain the same cable length. For this reason the definition of the span length given in Equation 34 should be used for wind loading.
Support Forces

When determining the forces that the cable tensions produce at the supports, the effects of weight, temperature and wind have to be added together. In order of importance, the tensions produced by wind loading are the greatest, then comes the tensions produced by the weight loading and finally the tensions produced by the temperature effect. Let’s take a look at the tensions produced by wind. If we look at a force diagram for a given span we find that the wind forces are on a plane that is angled with the cable slope perpendicular to the vertical plane. Figure 10 shows the top view of the force components that are generated by wind loading.

\[ T_{W} \text{ is the wind tension as calculated by equation 41 and the angle } \theta_{W} \text{ is the tension angle and is calculated using equation 42. It is important to note that } T_{W,z} \text{ is the lateral load produced directly by the wind and is equal to half the wind load at each support. The lateral load is calculated as follows:} \]

\[ F_{W,z} = T_{W} \sin(\theta_{W}) \]  

Equation 44
$T_{W,xy}$ is the force component that is aligned with the cable slope and so it produces a force along the cable direction and force along the support direction. $T_{W,xy}$ is calculated as follows:

$$F_{W,xy} = T_w \cos(\theta_w)$$  \hspace{1cm} \text{Equation 45}$$

Figure 11 takes a look at the vertical plane and shows a lateral view of the span. The tensions produced by the weight of the cable will be different for each support when there is a vertical offset. The tensions produced by the wind on the cable will always be equal for each support because it is assumed that only the perpendicular component of the wind is acting on the cable.

![Figure 11 – Wind and Weight Tension Components Lateral View](image)

As previously mentioned, the $T_{W,xy}$ component of the wind load is acting on a plane that is sloped with respect to the horizontal. The angle ($\gamma$) at which this tension is acting is calculated using equation 35. The x and y components of this force are calculated as follows:

$$F_{W,x} = T_{W,xy} \cos(\gamma)$$ \hspace{1cm} \text{Equation 46}$$

$$F_{W,y} = T_{W,xy} \sin(\gamma)$$ \hspace{1cm} \text{Equation 47}$$

$T_{w1}$ is the tension produced on support 1 by the weight load. The tension and angles are calculated using equation 19 and equation 23 for the parabolic cable or for the catenary cable as follows:

$$T_{w1} = T_D \cosh \left( \frac{-H}{C} \right) \quad \theta_1 = \tan^{-1} \left( \sinh \left( \frac{-H}{C} \right) \right)$$ \hspace{1cm} \text{Equation 48}$$

$T_{w2}$ is the tension produced on support 2 by the weight load. The tension and angle are calculated using equation 20 and equation 24 for the parabolic cable or for the catenary cable as follows:
\[ T_{w1} = T_D \cosh\left(\frac{Ax - H}{C}\right) \quad \theta_2 = \tan^{-1}\left(\sinh\left(\frac{Ax - H}{C}\right)\right) \quad \text{Equation 49} \]

The x and y components of the tension produced by weight acting on support 1 are calculated as follows:

\[ F_{w1,x} = T_{w1} \cos(\theta_1) \quad \text{Equation 50} \]
\[ F_{w1,y} = T_{w1} \sin(\theta_1) \quad \text{Equation 51} \]

The x and y components of the tension produced by weight acting on support 2 are calculated as follows:

\[ F_{w2,x} = T_{w2} \cos(\theta_2) \quad \text{Equation 50} \]
\[ F_{w2,y} = T_{w2} \sin(\theta_2) \quad \text{Equation 51} \]

Three forces will be acting in each support that produce a moment, \( T_{W,z}, T_{W,x} \) and \( T_{w,x} \). The wind tension was calculated as if the wind would be acting on a plane perpendicular to the vertical plane along the cable. This is the direction that the wind will produce the greatest tension and is needed to verify that the cable will withstand the maximum tensions produced by the wind. If the support has cables coming from different angles, then the wind will be acting at different angles on each line and using the maximum wind tension in each cable will over-design the support. It should be best if the wind direction is iterated and the tension produced by the wind on the cables is only the perpendicular component. PREPA guidelines suggest this iteration on the wind direction, but only for the lateral wind load (\( T_{W,z} \)).

**Cable Calculations**

Assuming that the geometric, cable and support data is given, the first step in the process of designing the aerial lines is to calculate the parameters that defined the shape of the cable based on its linear weight for 90°F and check that the cable complies with the required clearances and cable strength. This calculation is representative of what happens when the cable is installed. The order of the calculations is as follows:

1. Assume a value for \( T_{D,90°F} \), the cable lowest tension.
2. Calculate the catenary \( C_{90°F} \) Factor using equation 2.
3. Calculate the catenary \( H_{90°F} \) offset. This requires an iteration of \( H \) using equation 4.
4. Calculate the catenary \( V_{90°F} \) offset using equation 3.
5. Draw the cable profile using equation 1 and verify that the required separation from ground or roads comply with PREPA guidelines.
6. It is useful to calculate the sag, such that this information can be used for line installation. To determine the SAG first we need to know where the SAG occurs. This is done by iterating \( X_{SAG,90°F} \) using equation 8.
7. Calculate the SAG\( _{90°F} \) using equation 9.
8. Calculate the tensions $T_{w1,90°F}$ and $T_{w2,90°F}$ and angles $\theta_{w1,90°F}$ and $\theta_{w2,90°F}$ produced by the cable weight on the supports using equations 48 and 49.

9. If ground clearance or SAG is not met, go back to step 1 and increase $T_{D,90°F}$ to increase clearance (decrease the SAG) or decrease $T_{D,90°F}$ to decrease clearance (increase the SAG).

10. Calculate the length of the cable $L_{w,90°F}$ using equation 11.

The next step is to calculate the parameters that define the shape of the cable based on its linear weight for 60°F and 212°F as required by PREPA guidelines. Verification of the ground clearances should be done with 212°F calculations as the cable SAG will increase. Verification of the cable strength should be done with the 60°F as the tensions will increase. These calculations are representative of what happens when the cable is in operation. The order of the calculations is as follows:

1. Calculate the cable length $L_{w,60°F}$ and $L_{w,212°F}$ using the coefficient of expansion.
2. Calculate the cable linear weight $q_{w,60°F}$ and $q_{w,212°F}$ using the $L_{w,60°F}$ and $L_{w,212°F}$ and the 90°F cable weight ($q_{w,90°F} \times L_{w,90°F}$) as follows:

$$q_{w,60°F} = \frac{q_{w,90°F} \times L_{w,90°F}}{L_{w,60°F}}$$

Equation 52

$$q_{w,212°F} = \frac{q_{w,90°F} \times L_{w,90°F}}{L_{w,212°F}}$$

Equation 53

3. Calculate the $T_{D,60°F}$, $T_{D,212°F}$ that satisfies the cable length at each temperature. This is accomplished iterating on $T_D$ using equation 11. A value for the $C_{60°F}$ and $C_{212°F}$ must be calculated using equation 2 and a value of $H_{60°F}$ and $H_{212°F}$ shall be calculated iterating on $H$ using equation 4 for each iteration of $T_D$.

4. Calculate the catenary $V_{60°F}$ and $V_{212°F}$ offset using equation 3.

5. Draw the cable profile using equation 1 and verify that the required separation from ground or roads is as per PREPA guidelines. In this case, 20 Ft. separation is required.

6. It is useful to calculate the SAG, such that this information can be used for line operating conditions. To determine the SAG first we need to know where the SAG occurs. This is done by iterating $X_{SAG,60°F}$ and $X_{SAG,212°F}$ using equation 8.

7. Calculate SAG$_{60°F}$ and SAG$_{212°F}$ using equation 9.

8. Calculate the tensions $T_{w1,60°F}$, $T_{w2,60°F}$, $T_{w1,212°F}$ and $T_{w2,212°F}$ produced by the cable weight on the supports using equations 48a and 49a.

9. Calculate the angles $\theta_{w1,60°F}$, $\theta_{w2,60°F}$, $\theta_{w1,212°F}$ and $\theta_{w2,212°F}$ produced by the cable weight on the supports using equations 48b and 49b.

10. If ground clearance or SAG is not met on the 212°F calculations, go back to step 1 on the 90°F and increase $T_{D,90°F}$ to increase clearance (decrease the SAG) or decrease $T_{D,90°F}$ to decrease clearance (increase the SAG).

The above mentioned calculations were done using the catenary solution, but the parabolic approximation could have been used following a similar process. Now that we have solutions...
that satisfy normal installation and operation, calculations for the operation during wind loading
are done as follows:

1. Calculate the angled span length $\Delta l$ using equation 34.
2. Calculate the angle $\gamma$ between the cable start and end using equation 35.
3. Calculate the wind distributed load $q_{WIND}$ using equation 36.
4. Calculate the lowest tension $T_{D, WIND}$. This is accomplished iterating on $T_{D, WIND}$ using equation 43. The parabolic factor $K_{WIND}$ shall be calculated using equation 13 on each iteration. The length of the cable to be used will depend on the length of the cable for the selected temperature. Because lower temperatures than normal decrease cable length and increase tensions, the 60°F cable length is used because it is the case with the highest tensions in the cable.
5. Calculate the parabolic factor $K_{WIND}$ using equation 13.
6. Calculate the wind SAG $W_{WIND}$ using equation 38. This SAG $W_{WIND}$ can be compared against the right of way for the line to check compliance with PREPA guidelines. If SAG $W_{WIND}$ is greater than the right of way, calculations have to be run from the beginning for 90°F using a lower $T_{D, 90°F}$.
7. Calculate the wind tension $T_W$ and angle $\theta_W$ using equation 41 and 42.
8. Calculate the total cable tension $T_1$ and $T_2$ using the following equation:

\[
T_1 = \sqrt{\frac{T_{w_1}^2 + T_W^2}{2}} \quad \text{Equation 54}
\]

\[
T_2 = \sqrt{\frac{T_{w_2}^2 + T_W^2}{2}} \quad \text{Equation 55}
\]

9. If the total cable tensions at the supports exceed the cable strength, go back to the beginning and decrease $T_{D, 90°F}$. Tension check shall be done using a load factor of 0.60 on the cable ultimate strength.

**Support Calculations**

To design the supports, the three forces that will produce moments in the support $T_{W,z}$, $T_{W,x}$ and $T_{w,x}$ will have to be calculated. These can be calculated solving for equations 44 through 51. Also the force of the wind itself on the support has to be calculated. These forces will have to be multiplied by the force acting arm to calculate the moment in the support. This total moment will then have to be multiplied a load factor of 1.6 and compared against the support moment capacity. Any excess moment will have to be taken care by guy wires. Also, a support with larger moment capacity can be used. Care must be taken to use only the lateral wind component based on the assumed wind angle. The aerial lines calculations shall be done for every different type of cable that is hanging from the support.

**Case Study**

The following case study is from a project where the power to feed the facility had to come from an offsite location and was in a mountainous area. The project required a 38KV line using (3) 3/0 cables and (1) 3#6 for neutral. The cables were to pass above a concrete spill way and had to
maintain the 20 ft. minimum clearance required by PREPA guidelines with a right of way of 50Ft. Two 60-H6 reinforced concrete poles were used. Figure 12 shows a profile of the ground and the lowest cable profile. The data for the problem is as follows:

Geometric Data:

\[
\Delta x (m) : 110.00 \\
\Delta y (m) : 29.53
\]

Cable Data:

<table>
<thead>
<tr>
<th>Tag</th>
<th>Type</th>
<th>( q ) (Lb/Ft)</th>
<th>( d ) (in)</th>
<th>( \alpha ) (1/°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>3/0</td>
<td>0.23</td>
<td>0.447</td>
<td>1.46x10^{-5}</td>
</tr>
<tr>
<td>T2</td>
<td>3#6</td>
<td>0.23</td>
<td>0.447</td>
<td>1.46x10^{-5}</td>
</tr>
</tbody>
</table>

Support Data:

<table>
<thead>
<tr>
<th>Tag</th>
<th>Type</th>
<th>Height (Ft)</th>
<th>Height Above Ground (Ft)</th>
<th>Base Width (in)</th>
<th>Top Width (in)</th>
<th>Max. Moment (Lb-Ft)</th>
<th>Cable Separations (Ft)</th>
<th>Lowest Line Height (Ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P21</td>
<td>POLE 60H6</td>
<td>60</td>
<td>51.6</td>
<td>19.3</td>
<td>11.0</td>
<td>583296.0</td>
<td>0.5, 4.5, 8.5, 12.5</td>
<td>39.1</td>
</tr>
<tr>
<td>P22</td>
<td>POLE 60H6</td>
<td>60</td>
<td>51.6</td>
<td>19.3</td>
<td>11.0</td>
<td>583296.0</td>
<td></td>
<td>39.1</td>
</tr>
</tbody>
</table>

Figure 12 – Case Study Cable and Ground Profile
Calculations for this problem were done using a spreadsheet in Microsoft Excel® with the iterative solutions programmed in Visual Basic® macros. The solution to all the iterative calculations required less than a second making feasible to manually iterate on the value of $T_{D,90°F}$. Table 1 shows the results for the weight loading on Cable T-1.

Table 1 – Case Study Weight Calculations for Cable T-1

<table>
<thead>
<tr>
<th>Weight Calculations – Cable T-1</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°F):</td>
<td>90.0</td>
<td>60.0</td>
<td>212.0</td>
</tr>
<tr>
<td>Vertex Tension (Lb.):</td>
<td>215.6</td>
<td>224.3</td>
<td>194.0</td>
</tr>
<tr>
<td>Cable Length (Ft):</td>
<td>375.73</td>
<td>375.57</td>
<td>376.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equation Calculations - Catenary</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C Factor</td>
<td>285.7900</td>
<td>297.3326</td>
<td>257.3228</td>
</tr>
<tr>
<td>Horizontal Offset (m):</td>
<td>-20.3749</td>
<td>-23.4560</td>
<td>-12.7724</td>
</tr>
<tr>
<td>Vertical Offset (m):</td>
<td>-286.5166</td>
<td>-298.2583</td>
<td>-257.6398</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sag Calculations - Catenary</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sag Offset (m):</td>
<td>55.456</td>
<td>55.438</td>
<td>55.505</td>
</tr>
<tr>
<td>Sag (m):</td>
<td>5.495</td>
<td>5.280</td>
<td>6.107</td>
</tr>
<tr>
<td>Sag (Ft):</td>
<td>18.023</td>
<td>17.320</td>
<td>20.030</td>
</tr>
<tr>
<td>Sag (%):</td>
<td>5.00%</td>
<td>4.80%</td>
<td>5.55%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tension (Lb.) - Catenary</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P21</td>
<td>216.1</td>
<td>225.0</td>
<td>194.3</td>
</tr>
<tr>
<td>P22</td>
<td>238.4</td>
<td>247.3</td>
<td>216.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angle - Catenary</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P21</td>
<td>4.08</td>
<td>4.52</td>
<td>2.84</td>
</tr>
<tr>
<td>P22</td>
<td>25.28</td>
<td>24.89</td>
<td>26.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight Forces Along Cable Direction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P21</td>
<td>215.6</td>
<td>224.3</td>
<td>194.0</td>
</tr>
<tr>
<td>P22</td>
<td>-215.6</td>
<td>-224.3</td>
<td>-194.0</td>
</tr>
<tr>
<td>Check Sum</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Weight Forces Along Support

| P21 | 15.4 | 17.7 | 9.6 |
| P22 | -101.8 | -104.1 | -96.1 |

Sum of Forces: -86.4 -86.4 -86.5

Cable Weight: 86.4 86.4 86.5

Check Sum: 0.0 0.0 0.0

These calculations were also done for Cable T-2, taking care that because line T-2 is the highest (neutral) the sag is equal or less than that of T-1. It is important to note that these calculations show a balance of forces in all directions. The calculations also show that the behavior of the temperature is as expected. The largest sag occurs for 212°F and the largest tension occurs for 60°F. The tensions produced due to the weight of the cable for sags close to 5% are not large when compared to the cable strength. It is important to see how the calculated shape parameters provide us with a profile of the cable at the different temperatures. Figure 13 shows the profile of Cable T-1. Again the profile was generated using a spreadsheet in Microsoft Excel®. Data of the ground elevations was also included in the spreadsheet to check for clearances.
Wind calculations were also part of the spreadsheet where the weight calculations were done. In this way the solutions obtained for the weight loading were used to calculate the wind loads. The basic assumption here is that the solution to the wind load is coupled to the weight load solution because the length of the cable must remain equal for both cases. The spreadsheet prepared does the wind calculations for the three design temperatures as well. Again, the rationale was to check for tensions using the 60°F calculations and the right of way with the 212°F calculations. Table 2 shows the wind calculations for Cable T-1.

Looking at the lateral sag caused by the wind in a cable operating at a temperature of 212°F, one can see that it is close to 20 ft. Wind sags are smaller than weight sags, this is mainly because with the angled span length $\Delta l$ larger than the horizontal span $\Delta x$, the only way to maintain the same cable length is by making the sag smaller. The right of way for 38KV cable above ground is 50 ft. The lateral sag then complies with the 25 ft. right of way left on each side of the cable route. If the lateral sag of the cable is not taken care of in the design by keeping the sag percentage small (<5%), exceeding the right of way distance is possible. In the case of a structure close to the right of way, wind loading could cause the cable to get close enough to the structure causing an electric arc and endangering the people inside the building. Again, it is important to note that these calculations show a balance of forces for all directions. If the tensions produced by the wind are compared to those produced by the weight one can see that wind loads are more than 7 times those of the weight. This ratio gets even higher with higher supports because ASCE 7-95 increases the wind pressure depending on height. Cable T-2 height is 12 feet above the lowest cable for this case study. This made the wind pressure be increased from 45 PSF to 50 PSF. Table 3 shows the Total Tension calculations for Cable T-1. As it can be seen, the cable strength is not exceeded.
Table 2 – Wind Calculations for Cable T-1

<table>
<thead>
<tr>
<th>Wind Calculations</th>
<th>90.0</th>
<th>60.0</th>
<th>212.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°F):</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wind Load (Lb/Ft):</td>
<td>1.6763</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angled Span (Ft):</td>
<td>373.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertex Tension (Lb):</td>
<td>1678.6</td>
<td>1746.9</td>
<td>1510.1</td>
</tr>
</tbody>
</table>

| Equation Calculations – Parabolic-Symmetric            |       |       |       |
| Factor:                                                | 1.6377E-03 | 1.5737E-03 | 1.8204E-03 |
| Horizontal Offset (m):                                 | 56.9475 | 56.9475 | 56.9475 |
| Vertical Offset (m):                                   | -5.3110 | -5.1034 | -5.1034 |

| Sag Calculations - Parabolic-Symmetric                 |       |       |       |
| Sag Offset (m):                                        | 56.95 | 56.95 | 56.95 |
| Sag (m):                                               | 5.31 | 5.10 | 5.90 |
| Sag (Ft):                                              | 17.42 | 16.74 | 19.36 |
| Sag (%):                                               | 4.83% | 4.64% | 5.37% |

| Tension (LB.) – Parabolic-Symmetric                    |       |       |       |
| P21                                                    | 1707.57 | 1774.76 | 1542.24 |
| P22                                                    | 1707.57 | 1774.76 | 1542.24 |

| Angle – Parabolic-Symmetric                            |       |       |       |
| P21                                                    | -10.57 | -10.16 | -11.71 |
| P22                                                    | 10.57 | 10.16 | 11.71 |

| Wind Forces Along Cable                                |       |       |       |
| P21                                                    | 1678.62 | 1746.93 | 1510.12 |
| P22                                                    | -1678.62 | -1746.93 | -1510.12 |
| Check Sum                                             | 0.00 | 0.00 | 0.00 |

| Wind Forces Along Cable Direction                      |       |       |       |
| P21                                                    | 1621.22 | 1687.18 | 1458.48 |
| P22                                                    | -1621.22 | -1687.18 | -1458.48 |
| Check Sum                                             | 0.0 | 0.0 | 0.0 |

| Wind Forces Along Support                              |       |       |       |
| P21                                                    | 435.24 | 452.95 | 391.55 |
| P22                                                    | -435.24 | -452.95 | -391.55 |
| Check Sum                                             | 0.00 | 0.00 | 0.00 |

| Wind Forces Perpendicular to Cable                     |       |       |       |
| P21                                                    | -313.1 | -313.1 | -313.1 |
| P22                                                    | -313.1 | -313.1 | -313.1 |
| Sum Of Forces:                                         | -626.2 | -626.2 | -626.2 |
| Wind Force:                                            | 626.2 | 626.2 | 626.2 |
| Check Sum                                             | 0.0 | 0.0 | 0.0 |
Table 3 – Total Forces Calculations for Cable T-1

<table>
<thead>
<tr>
<th>Total Forces Along Cable Direction:</th>
<th>Temperature (°F):</th>
<th>90.0</th>
<th>60.0</th>
<th>212.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>P21</td>
<td>1836.8</td>
<td>1911.5</td>
<td>1652.5</td>
<td></td>
</tr>
<tr>
<td>P22</td>
<td>-1836.8</td>
<td>-1911.5</td>
<td>-1652.5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total Forces Perpendicular to Cable Direction</th>
<th>P21</th>
<th>-313.1</th>
<th>-313.1</th>
<th>-313.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>P22</td>
<td>-313.1</td>
<td>-313.1</td>
<td>-313.1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total Forces Along Support Direction</th>
<th>P21</th>
<th>450.6</th>
<th>470.7</th>
<th>401.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P22</td>
<td>-537.0</td>
<td>-557.1</td>
<td>-487.7</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cable Total Tension (Lb.):</th>
<th>P21</th>
<th>1721.2</th>
<th>1789.0</th>
<th>1554.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>P22</td>
<td>1724.1</td>
<td>1791.9</td>
<td>1557.4</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cable Tension Check</th>
<th>Ultimate Strength (Lb):</th>
<th>6620</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Load Factor:</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>Working Load(Lb):</td>
<td>3972</td>
</tr>
<tr>
<td></td>
<td>Tension Check:</td>
<td>OK</td>
</tr>
</tbody>
</table>

Finally let’s look at the support calculations. Because supports can have different cables attached to them with different heights and sizes, the calculations of supports were done on spreadsheets separate from the cable calculations. Nonetheless, the information between them was coupled. In order to get the height of the lines, the cable spreadsheets require the information of support heights from the supports calculations. Coupling of the spreadsheets was accomplished by means of the Cable and Pole tags rather than cell directions. This allows the construction of a system of poles and cables in a simple manner.

Table 4 – Support Calculations for Support P21

<table>
<thead>
<tr>
<th>Support Forces and Moments</th>
<th>Cable Tag</th>
<th>Cable Offset (Ft)</th>
<th>Cable Height (Ft)</th>
<th>Long. Force (Lb)</th>
<th>Lateral Load (Lb)</th>
<th>Support Load (Lb)</th>
<th>Forces X (Lb)</th>
<th>Forces Y(Lb)</th>
<th>Resulting Force (Lb)</th>
<th>Resulting Moment (Lb-Ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T2</td>
<td>0.50</td>
<td>51.10</td>
<td>1605.1</td>
<td>-271.6</td>
<td>398.0</td>
<td>1605.1</td>
<td>271.6</td>
<td>1628.0</td>
<td>83189</td>
<td>83189</td>
</tr>
<tr>
<td>T1</td>
<td>4.50</td>
<td>47.10</td>
<td>1911.5</td>
<td>-313.1</td>
<td>470.7</td>
<td>1911.5</td>
<td>313.1</td>
<td>1937.0</td>
<td>91232</td>
<td>91232</td>
</tr>
<tr>
<td>T1</td>
<td>8.50</td>
<td>43.10</td>
<td>1911.5</td>
<td>-313.1</td>
<td>470.7</td>
<td>1911.5</td>
<td>313.1</td>
<td>1937.0</td>
<td>83484</td>
<td>83484</td>
</tr>
<tr>
<td>T1</td>
<td>12.50</td>
<td>39.10</td>
<td>1911.5</td>
<td>-313.1</td>
<td>470.7</td>
<td>1911.5</td>
<td>313.1</td>
<td>1937.0</td>
<td>75736</td>
<td>75736</td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7438.9</td>
<td>7438.9</td>
</tr>
</tbody>
</table>

The resulting moment has to be added to the moment produced by the wind in the support. This in turn has to be multiplied by the load factor of 1.6. The resulting moment to be resisted by the support is 692,578 Lb-Ft. Since the support capacity is 583,296 Lb-Ft., guy wires have to be used to take care of the excess moment. In this case the assumed wind direction was perpendicular to the cable direction. If other cables were attached to these supports, an iteration
of the wind angle would be necessary to obtain the maximum load at each support. Such iteration can be done manually with the spreadsheet prepared.

Conclusions

The preceding article shows the general formulation for the solution of the flexible cables hanging from supports using the catenary. The solutions presented show how to calculate the shape of the cables under weight load, the tension vector at the supports, the cable sag and the effects of temperature. In cases where the cable starting elevation is different from the ending elevation, it was shown that the catenary solution should be used to avoid errors in the calculated tensions produced by the weight of the cable. Using the parabolic solution, an approximation of the catenary was developed. This solution was adapted to the case of wind loading where a symmetric parabolic solution is required. It was shown that the correct span length to use in the wind loading calculations is the angled span length. It was shown that for small sag percentages (<6%) the calculated tensions using the horizontal span can yield significant errors. A case study was presented to show that the calculations are doable in a spreadsheet program. Checksum of forces showed the integrity of the equations derived.

The importance of having a complete viewpoint of the aerial cables design was shown. Such viewpoint has not been completely available to the engineering community in Puerto Rico. It is expected that engineering firms in Puerto Rico realize the importance of doing these calculations for the design of aerial power cables. Situations such as those lived during past weather emergencies should be avoided. A stringer protocol for the installation of power cables should be implemented. Such a protocol would call for calculation of support loads due to field changes to any aerial power distribution system.

References


Cruz, Héctor G.; Nuevos Criterios de Diseño para Líneas Eléctricas Aéreas; Autoridad de Energía Eléctrica División de Distribución Eléctrica, San Juan, P.R.; 2000.